Trigonométrie

1. $\forall x \in \mathbb{R} : \cos^2 x + \sin^2 x = 1$ $\quad \forall x \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\} : 1 + tg^2 x = \frac{1}{\cos^2 x}$

   $\forall x \in \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\} : 1 + \cot g^2 x = \frac{1}{\sin^2 x}$.

   - $\forall x \in \mathbb{R} : \cos(x + 2k\pi) = \cos x$; $\forall x \in \mathbb{R} : \sin(x + 2k\pi) = \sin x$
   - $\forall x \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi; k \in \mathbb{Z}\right\} : tg(x + k'\pi) = tgx; \ (k' \in \mathbb{Z})$
   - $\forall x \in \mathbb{R} \setminus \{k\pi; k \in \mathbb{Z}\} : cotg(x + k'\pi) = cotgx; \ (k' \in \mathbb{Z})$
   - $\forall x \in \mathbb{R} \setminus \left\{\frac{k\pi}{2}, k \in \mathbb{Z}\right\} : tgx \times \cot gx = 1$

2. Formules relatives aux angles associés :

$\cos(-x) = \cos x \ \forall x \in \mathbb{R}$	$\sin(-x) = -\sin x \ \forall x \in \mathbb{R}$	$tg(-x) = -tgx$ $\forall x \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$	$\cot g(-x) = -\cot gx$ $\forall x \in \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}$
$\cos(\pi - x) = -\cos x \ \forall x \in \mathbb{R}$	$\sin(\pi - x) = \sin x \ \forall x \in \mathbb{R}$	$tg(\pi - x) = -tgx$ $\forall x \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi; k \in \mathbb{Z}\right\}$	$\cot g(\pi - x) = -\cot gx$ $\forall x \in \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}$
$\cos(\pi + x) = -\cos x \ \forall x \in \mathbb{R}$	$\sin(\pi + x) = -\sin x \ \forall x \in \mathbb{R}$	$tg(\pi + x) = tgx$ $\forall x \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi; k \in \mathbb{Z}\right\}$	$\cot g(\pi + x) = \cot gx$ $\forall x \in \mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}$
$\cos(\frac{\pi}{2} - x) = \sin x \ \forall x \in \mathbb{R}$	$\sin(\frac{\pi}{2} - x) = \cos x \ \forall x \in \mathbb{R}$	$tg(\frac{\pi}{2} - x) = cotgx$ $\forall x \in \mathbb{R} \setminus \{k\pi; k \in \mathbb{Z}\}$	$\cot g(\frac{\pi}{2} - x) = tgx$ $\forall x \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$
$\cos(\frac{\pi}{2} + x) = -\sin x \ \forall x \in \mathbb{R}$	$\sin(\frac{\pi}{2} + x) = \cos x \ \forall x \in \mathbb{R}$	$tg(\frac{\pi}{2} + x) = -cotgx$ $\forall x \in \mathbb{R} \setminus \{k\pi; k \in \mathbb{Z}\}$	$\cot g(\frac{\pi}{2} + x) = -tgx$ $\forall x \in \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$

3. Formules d'addition :

   - $\cos(a + b) = \cos a \times \cos b - \sin a \times \sin b \ \ \forall a \in \mathbb{R} \ \forall b \in \mathbb{R}$
     $\cos(a - b) = \cos a \times \cos b + \sin a \times \sin b \ \ \forall a \in \mathbb{R} \ \forall b \in \mathbb{R}$
   - $\sin(a + b) = \sin a \times \cos b + \cos a \times \sin b \ \ \forall a \in \mathbb{R} \ \forall b \in \mathbb{R}$
     $\sin(a - b) = \sin a \times \cos b - \cos a \times \sin b \ \ \forall a \in \mathbb{R} \ \forall b \in \mathbb{R}$
   - $tg(a + b) = \frac{tga + tgb}{1 - tga \times tgb} \ \ \forall a \in D, \forall b \in D, \forall (a + b) \in D : D = \mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$

- $\operatorname{tg}(a-b)=\dfrac{\operatorname{tg}a-\operatorname{tg}b}{1+\operatorname{tg}a\times\operatorname{tg}b}\ \forall a\in D,\forall b\in D,\forall(a-b)\in D : D=\mathbb{R}\setminus\left\{\dfrac{\pi}{2}+k\pi,k\in\mathbb{Z}\right\}$

4. Formules de multiplication par 2 :
   - $\cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x\quad \forall x\in\mathbb{R}.$
   - $\sin 2x=2\sin x\times\cos x\ \forall x\in\mathbb{R}$
   - $\operatorname{tg}2x=\dfrac{2\operatorname{tg}x}{1-\operatorname{tg}^2x}\ ;(x\in\mathbb{R}\setminus\left\{\dfrac{\pi}{2}+k\pi,k\in\mathbb{Z}\right\};2x\in\mathbb{R}\setminus\left\{\dfrac{\pi}{2}+k\pi,k\in\mathbb{Z}\right\})$

5. Formules de transformation de sommes en produits :
   - $\cos p+\cos q=2\cos\dfrac{p+q}{2}\times\cos\dfrac{p-q}{2},\forall p\in\mathbb{R},\forall q\in\mathbb{R}$
   - $\cos p-\cos q=-2\sin\dfrac{p+q}{2}\times\sin\dfrac{p-q}{2},\forall p\in\mathbb{R},\forall q\in\mathbb{R}$
   - $\sin p+\sin q=2\sin\dfrac{p+q}{2}\times\cos\dfrac{p-q}{2},\forall p\in\mathbb{R},\forall q\in\mathbb{R}$
   - $\sin p-\sin q=2\cos\dfrac{p+q}{2}\times\sin\dfrac{p-q}{2},\forall p\in\mathbb{R},\forall q\in\mathbb{R}$

6. Formules de transformation de produits en sommes :
   - $\cos a\times\cos b=\dfrac{1}{2}[\cos(a+b)+\cos(a-b)]$
   - $\sin a\times\sin b=-\dfrac{1}{2}[\cos(a+b)-\cos(a-b)]$
   - $\sin a\times\cos b=-\dfrac{1}{2}[\sin(a+b)+\sin(a-b)]$

7. transformation de $a\times\cos x+b\times\sin x$:

$a\cos x+b\sin x=r\cos(x-\varphi)\ ;\ r=\sqrt{a^2+b^2};\cos\varphi=\dfrac{a}{r};\sin\varphi=\dfrac{b}{r};(a\in\mathbb{R}^*,b\in\mathbb{R}^*)$

8. Si on pose : $t=\operatorname{tg}\dfrac{x}{2}$ alors $\cos x=\dfrac{1-t^2}{1+t^2}$; $\sin x=\dfrac{2\times t}{1+t^2}$; $\operatorname{tg}x=\dfrac{2\times t}{1-t^2}$

9. Equations trigonométriques :
   - $\cos x=\cos\alpha\Leftrightarrow\begin{cases}x=\alpha+2k\pi,k\in\mathbb{Z}\\ \text{ou}\\ x=-\alpha+2k\pi,k\in\mathbb{Z}\end{cases}$
   - $\sin x=\sin\alpha\Leftrightarrow\begin{cases}x=\alpha+2k\pi,k\in\mathbb{Z}\\ \text{ou}\\ x=\pi-\alpha+2k\pi,k\in\mathbb{Z}\end{cases}$
   - $\operatorname{tg}x=\operatorname{tg}y\Leftrightarrow x=y+k\pi;k\in\mathbb{Z}.$
   - $\alpha\in\mathbb{R}\setminus\left\{\dfrac{\pi}{2}+k\pi,k\in\mathbb{Z}\right\};x\in\mathbb{R}\setminus\left\{\dfrac{\pi}{2}+k\pi,k\in\mathbb{Z}\right\}:\operatorname{tg}x=\operatorname{tg}\alpha\Leftrightarrow x=\alpha+k'\pi,k'\in\mathbb{Z}$
   - $\cos x=0\Leftrightarrow x=\dfrac{\pi}{2}+k\pi,k\in\mathbb{Z}$
   - $\sin x=0\Leftrightarrow x=k\pi,k\in\mathbb{Z}$
   - $\cos x=1\Leftrightarrow x=2k\pi,k\in\mathbb{Z}$